22th December: National Mathematics Day Special

Srinivasa Ramanujan: First Youngest FRS of the World

Srinivasa Ramanujan, FRS

Today is the birthday of Srinivasa Ramanujan Iyengar, a largely self-taught pure mathematician in the world. It is surprising to see how man has succeeded in becoming one of the most important pioneers in history, as have Newton and Einstein.

Srinivasa Ramanujan was born in a poor Brahmins family on December 22, 1887 in the town of Erode, in Tamil Nadu, in the south east of India.

Family :

Srinivasa Ramanujan's father was K. Srinivasa Iyengar, an accounting clerk for a clothing merchant. His mother was Komalatammal, who earned a small amount of money each month as a singer at the local temple. In July 1909, Ramanujan married S. Janaki Ammal, who was then just 10 years old.

Parents of Ramanujan

The School Life:

The young genius didn't like attending school. He didn't have friends as few could understand him. Leaving aside sports, Ramanujan picked up Math and lost himself in the numbers. Ramanujan's parents moved around a lot so he attended a variety of different elementary schools.

At the age of 11, he met two students who stayed with him for some time and he took advantage of this to learn all he could from them.

At the age of 12, he got a famous mathematical book of S. L. Linney's Plane Trigonometry from a senior school student. After complete reading of this book, he had begun serious self-study of mathematics, working through cubic equations, arithmetic and geometric series. He invented his own method of solving quartic equations.

The College life:

In 1904, Ramanujan left high school. He had won a scholarship which allowed him to study at the Government Arts College in the town of Kumbakonam, but with a great interest in mathematics, Ramanujan failed his non-mathematical exams and lost his scholarship.

In 1905, he traveled to Madras and enrolled at Pachaiyappa's College, but again failed his non-mathematical exams.

Inspiration from book:

At the age of 16, Ramanujan read a library loaned copy from his friend called "A Synopsis of

Elementary Results in Pure and Applied Mathematics" by G. S. Carr. This book has 5000 theorems. This is a book that is said to have truly awakened his seer-like genius. So this book became the main source of inspiration and expertise.

At the age of 19, Ramanujan couldn't use paper as it was quite expensive. He worked on his derivations on slate, so he noted down only the important results and summaries in his notebooks. The mathematics he was doing was very highly original and very advanced.

He independently discovered results originally discovered by some of the greatest mathematicians in history, such as Carl Friedrich Gauss and Leonhard Euler.

First Job:

At the age of 22 (year 1910), Ramanujan realized he must find work to stay alive. In the city of Madras he found some students who needed mathematics tutoring and he also walked around the city offering to do accounting work for businesses.

Ramanujan got a job as clerk in the government revenue department of the Madras Port Trust and where he met an officer whose name was Ramaswamy Aiyer. Ramanujan had shown notebooks (the results of his mathematical work) as a resume to Ramaswamy Aiyer.

Ramaswamy Aiyer was surprised when he saw the mathematical work of Ramanujan. Luckily Ramaswamy Aiyer was a mathematician also. He had recently founded the Indian Mathematical Society. After that Ramanujan got the permission to do mathematics at the workplace after finishing his daily official work from the Madras Port Trust's Chief Accountant, S. Narayana Iyer, who was treasurer of the Indian Mathematical Society.

In 1911, Ramanujan's 17-page paper about Bernoulli numbers appeared in the Journal of the Indian Mathematical Society.

Ph.D at Cambridge:

At the age of 25, Ramanujan attended college hoping to pass the exam required to enter the University of Madras. But he was so absorbed in math that he ignored other subjects and failed in non-mathematical subjects. He never earned a bachelor's degree.

After two years, Ramanujan contacted experts in England to evaluate his work, but because of his lack of formal education, he was not taken seriously.

Ramanujan wrote a 10-page letter to a number of British professors with over 120 statements of theorems on infinite series, improper integrals, continued fractions and number theory without showing the details of how he had acquired the results. But only one was receptive – an eminent pure mathematician at the University of Cambridge – Godfrey Harold Hardy, known to everyone as G. H. Hardy, who received a letter from Ramanujan in January 1913.

Hardy reviewed the papers of Ramanujan with J. E. Littlewood, another eminent Cambridge mathematician. Hardy told Littlewood that the paper had been written by either a crank or a genius. Therefore, these renowned mathematicians took a look at Ramanujan's work and checked them as "fraud".

But after looking over the original work of Ramanujan in paper, Hardy and Littlewood came to a conclusion that they were looking at the papers of a mathematical genius.In February, 1913, Hardy invited Ramanujan to come to Cambridge for further study.

Prof. Hardy wrote in the letter that *"I had never seen anything like them before. A single look at*

them was enough to show that they could be written by a mathematician of the highest class. They must be true because, if they were not true, no one would have the imagination to invent them".

But there was opposition to Ramanujan's going abroad given his strict religious background. However, having been strictly brought up as a Brahmin, Ramanujan considered the culinary and cultural differences he would face; and combined with the vehement objections from his mother, Ramanujan refused to go.

It was only later, when her mother spoke about her living dream in which their family goddess Namagiri instructed her to no longer stand between her son and the fulfillment of his life's purpose after that Ramanujan agreed to go to Cambridge.

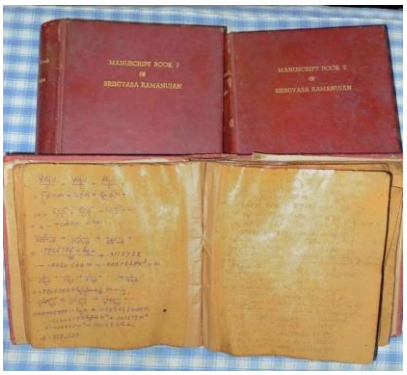
Ramanujan said the Hindu goddess Namagiri – who had appeared in his mother's dream telling her to allow him to go to Cambridge – had appeared in one of his own dreams.

Ramanujan arrived in Cambridge in April 1914, three months before the outbreak of World War 1. Within days he had begun work with Hardy and Littlewood.

Hardy and his colleague, J E Littlewood, went out of the way to admit Ramanujan to Cambridge University, despite his lack of a degree in mathematics.

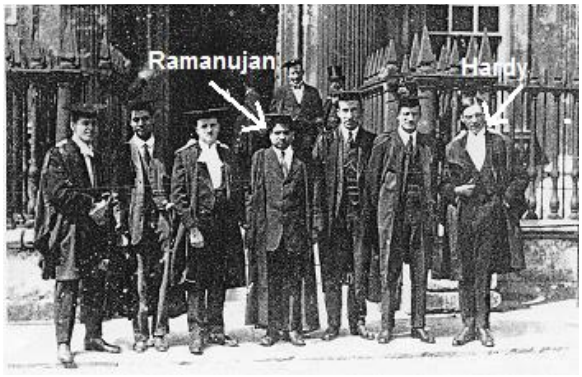
When Ramanujan left to study at the University of Cambridge, his wife moved with Ramanujan's parents. Ramanujan's scholarship was sufficient for his needs in Cambridge and the family's needs in Kumbakonam.

The notebooks he brought from India were filled with thousands of identities, equations, and theorems.



Ramanujan's manuscripts

He published many new results on topics such as the number theory, infinite series and indefinite integrals. One of the most spectacular results in mathematics is the Hardy-Ramanujan formula derived in 1917 for the number of partitions of an integer.



Srinivasa Ramanujan (middle) with fellow scientists at Cambridge

Srinivasa Ramanujan received an honorary bachelor's degree from Cambridge University in 1916 and in the same year he earned a Ph.D for his work on highly composite numbers.

Hardy had proposed the name of Ramanujan to be elected as the Fellow of the prestigious Royal Society, the first ever Indian scientist, that too at a very young age of 30. The proposal was to be taken up for consideration in an upcoming meeting of the society.

On 28th February, 1918 -Ramanujan, just 31-year-old became the second Indian Fellow of the Royal Society and the youngest in world history. The same year, he became the first Indian to be elected as a Fellow of Trinity College, Cambridge.

Famous incident about the Hardy-Ramanujan number:

One of the famous incidents that shows Ramanujan's love for numbers was when Hardy once met him in the hospital. When Hardy got there, he told Ramanujan that his cab's number, 1729, was "rather a dull number" and hoped it didn't turn out to be an unfavorable omen. To this, Ramanujan said, "No, it is a very interesting number. It is the smallest number expressible as a sum of two cubes in two different ways." .

The number "1729" is now known as the Hardy-Ramanujan number. It is the sum of the cubes of two numbers 10 and 9. This is the smallest number that can be expressed in two different ways as it is the sum of these two cubes. Interestingly, 1729 is a natural number following 1728 and preceding 1730.

Ramanujan's contributions to mathematics:

Ramanujan compiled around 3,900 results consisting of equations and identities. All the contributions made by Srinivasa Ramanujan and the related facts stand as verifiable proof that he reached unimaginable heights in his short life of little over 32 years and defined his own standard of genius. Ramanujan's contributions stretch across mathematics fields, including complex analysis, number theory, infinite series, and continued fractions. Other notable contributions by Ramanujan include hypergeometric series, the Riemann series, the elliptic integrals, the theory of divergent series, and the functional equations of the zeta function.

Infinite series for pi:

In 1914, in England, Ramanujan found a formula for infinite series for pi, which forms the basis of many algorithms used today. Finding an accurate approximation of π (pi) has been one of the most important challenges in the history of mathematics. However, the problem was that it was prolonged. For example, it took about 600 terms to come to a value of 3.14 but Ramanujan developed a series that would converge to 3.141592 just after one term.

Hardy-Ramanujan Asymptotic Formula:

One work of Ramanujan done with G. H. Hardy is his formula for the number of partitions of a positive integer n, the famous Hardy-Ramanujan Asymptotic Formula for the partition problem. The formula has been used first by Niels Bohr to calculate quantum partition functions of atomic nuclei. This is used in statistical physics also.

Ramanujan summation of numbers in series:

Ramanujan invented summation, which is a technique for assigning a value to divergent infinite series. It is essentially a property of the partial sums rather than a property of the entire sum. For example, this method for summation of numbers points to the fact that 'S'= -1/12, where S = 1+2+3+4+5+6+7+.....Though this result is shocking, string theory, quantum field theory, and some complex analytics extensively use this to derive equations.

Highly composite numbers:

A highly composite number is basically a positive integer that has more divisors than any smaller positive integer. Ramanujan coined this term in the year 1915. There is an infinite number of highly composite numbers, the first few being 1, 2, 4, 6, 12, 24, 36, 48, 60...and so on. The corresponding numbers of divisors are 1, 2, 3, 4, 6, 8, 9, 10, 12... and so on. In 1915, he listed 102 highly composite numbers up to 6746328388800.

In Signal processing:

Today, all the signals that are processed digitally comprise certain patterns that repeat over and over again. These signals can be anything like

that of speech, music, and even more, research-oriented ones such as DNA and protein sequences. While processing such signals, being able to extract and identify periodic information is of fundamental concern. To achieve this, the concept of "Ramanujan subspaces" is used.

To study the nature of Black Holes:

Nowadays, Ramanujan and Hardy's results on partitions and the mock theta functions have found an important role in understanding the very quantum structure of spacetime, particularly in the comprehension of quantum entropy of a type of Black Hole in string theory. Thus Ramanujan's otherwise simple mathematical calculation is now being utilized in revealing the complicated properties of black holes.

Theta function is a special function of several complex variables. German mathematician Carl Gustav Jacob Jacobi invented several closely related theta functions known as Jacobi theta functions. Theta function was studied extensively by Ramanujan who came up with the Ramanujan theta function, that generalizes the form of Jacobi theta functions and also captures general properties. Ramanujan theta function is used to determine the critical dimensions in Bosonic string theory, superstring theory, and M-theory.

$$f(a,b) = \sum_{n=-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}}$$

The Ramanujan theta function

Thoughts of Scientists about Ramanujan:

G. H. Hardy liked to rank mathematicians on a scale of 1 to 100, and he gave himself 25, Littlewood 30, David Hilbert 80, and Ramanujan 100, which shows just how great Ramanujan was.

"... each of the 24 modes in the Ramanujan function corresponds to a physical vibration of a string. Whenever the string executes its complex motions in space-time by splitting and recombining, a large number of highly sophisticated mathematical identities must be satisfied. These are precisely the mathematical identities discovered by Ramanujan."

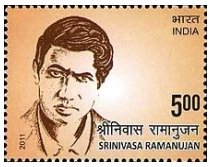
.....Theoretical physicist Michio Kaku

Notebook published as book:

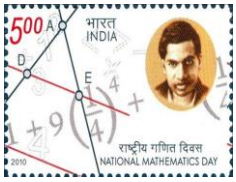
One of Ramanujan's notebooks was discovered by George Andrews in 1976 in the library at Trinity College. Professor Bruce C Berndt spent 22 years trying to prove the 3,254 theorems in Ramanujan's three notebooks and the "lost notebook" – a set of pages Janaki gathered together from their Indian home after his death. He has said that "some of Ramanujan's math is simply startling. If he had not discovered them, nobody ever would have."Later the contents of these notebooks were published as a book named "Notebooks of Srinivasa Ramanujan" and "Ramanujan's lost Notebook".

India Post released the Commemorative stamps (by year):

1962



2011



2012

2016

The winter weather in England, much colder than anything he had ever imagined, made him ill for a time. Ramanujan's insufficient diet in England during the time of war, while trying to maintain his staunch Brahmin eating habits, combined with his long hours of study and research gradually broke down his health. In 1917,

he was diagnosed with tuberculosis and worryingly low vitamin levels. Unfortunately Srinivasa Ramanujan died aged 32 in Madras on April 26, 1920. His death was most likely caused by hepatic amoebiasis caused by liver parasites common in Madras.

It is truly not often when you find such a simple mannered, unassuming man who breaks all stereotypes despite little formal education and no such exposure to advanced mathematics. Here's hoping that Srinivasa Ramanujan gets due credit for what he was the greatest mathematician in the world of his time. His birthday, 22 December, is celebrated as National Mathematics day in India.

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